

# Particle Statistics in Quantum Information Processing

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Particle statistics is a fundamental part of quantum physics, and yet its role and use in the context of quantum information have been poorly explored so far. After briefly introducing particle statistics and the Symmetrization Postulate, I will argue that this fundamental aspect of Nature can be seen as a resource for quantum information processing and I will present examples showing how it is possible to do useful and efficient quantum information processing using only the effects of particles statistics.

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## I. AN EXTRA POSTULATE IS REQUIRED

I believe most physicists would consider that the postulates (or at least the properties they embody) concerning the superposition, evolution and measurement of quantum states cover the essence of Quantum Mechanics, the theory that is at the basis of current fundamental Physics and gives us such an accurate description of Nature at the atomic scale. Yet, if the theory was only based on these postulates (or properties), its descriptive power would be almost zero and its interest, if any, would be mainly mathematical. As soon as one wants to describe matter, one has to include an extra postulate: *Pauli's Exclusion Principle*. One of its usual formulations, equivalent to the one proposed originally by Wolfgang Pauli in 1925 [1], is the following:

**Pauli's Exclusion Principle** — *No two electrons can share the same quantum numbers.*

This principle refers to electrons, which constitute a significant (but not the whole) part of matter, and is crucial in helping us explain a wide range of phenomena, including:

- The electronic structure of atoms and, as a consequence, the whole Periodic Table;
- The electronic structure of solids and their electrical and thermal properties;
- The formation of white dwarfs, where the gravitational collapse of the star is halted by the pressure resulting from its electrons being unable to occupy the same states;
- The repulsive force that is part of the *ionic bond* of molecules and puts a limit to how close the ions can get (e.g., 0.28 nm between  $Na^+$  and  $Cl^-$  for solid sodium chloride), given the restrictions to the states the overlapping electrons can share.

We thus see how Pauli's insight when proposing the Exclusion Principle was fundamental for the success of

Quantum Mechanics. Although he made many other important contributions to Physics, it was for this one that he was awarded the Nobel prize in 1945.

Pauli's Exclusion Principle remains as a postulate, for Pauli's own dissatisfaction, as he expressed in his Nobel prize acceptance lecture in 1946 [2]:

*"Already in my original paper I stressed the circumstance that I was unable to give a logical reason for the exclusion principle or to deduce it from more general assumptions. I had always the feeling, and I still have it today, that this is a deficiency."*

In any case, as inexplicable as it may be, Pauli's Exclusion Principle seems to beg for a generalization. In fact, it was soon realized that other particles apart from electrons suffer from the same inability to share a common quantum state (e.g., protons). More surprising was the indication that some particles seem to obey to the exactly opposite effect, being — under certain circumstances — forced to share a common state, as for instance photons in the stimulated emission phenomenon, thus calling for a much more drastic generalization of Pauli's Principle.

## II. IDENTITY AND INDISTINGUISHABILITY

Pauli's Exclusion Principle intervenes in a wide range of phenomena, from the chemical bond in the salt on our table to the formation of stars in distant galaxies. This is because it applies to electrons and we consider all electrons in the universe to be *identical*, as well as any other kind of quantum particles:

**Identical particles** — *Two particles are said to be identical if all their intrinsic properties (e.g., mass, electrical charge, spin, colour, ...) are exactly the same.*

Thus, not only all electrons are identical, but also all positrons, photons, protons, neutrons, up quarks, muon neutrinos, hydrogen atoms, etc. They each have the same defining properties and behave the same way under the interactions associated with those properties. This brings

us to yet another purely quantum effect, that of *indistinguishable* particles.

How can we distinguish identical particles? Their possibly different internal states are not a good criterion, as the dynamics can in general affect the internal degrees of freedom of the particles. The same is valid for their momentum or other dynamical variables. But their spatial location can actually be used to distinguish them. Let us imagine we have two identical particles, one in Alice's possession and the other with Bob. If these two parties are kept distant enough so that the wave functions of the particles practically never overlap (during the time we consider this system), then it is possible to keep track of the particles just by keeping track of the classical parties. This situation is not uncommon in quantum mechanics. If, on the other hand, the wave functions do overlap at some point, then we no longer know which particle is with which party. And if we just do not or cannot involve these classical parties at all, then it is in general also impossible to keep track of identical particles. In both these cases, the particles become completely indistinguishable, they are identified by completely arbitrary labels, with no physical meaning (as opposed to *Alice* and *Bob*). In these situations the description of our system becomes ambiguous and the so-called *exchange degeneracy* appears. The problem of finding the correct and unambiguous description for such systems is very general and requires the introduction of a new postulate for quantum mechanics: the Symmetrization Postulate.

**Symmetrization Postulate** — *In a system containing indistinguishable particles, the only possible states of the system are the ones described by vectors that are, with respect to permutations of the labels of those particles:*

- *either completely symmetrical — in which case the particles are called bosons;*
- *either completely antisymmetrical — in which case the particles are called fermions.*

This is in fact a generalization of Pauli's Exclusion Principle, in two ways. First, it extends it to a whole class of particles which suffer the same restrictions: fermions. But it goes even further and introduces a new class of particles, bosons, which have a very different behaviour, almost the opposite, as they are forced to share the same quantum numbers. To decide which particles should be associated to a particular symmetry is something that must ultimately be determined by observation. The Symmetrization Postulate matches the study of such symmetries with our empirical knowledge: as far as we know today, there are two classes of particles in Nature according to their collective behaviour in indistinguishable situations. These are, of course, bosons and fermions: no particles have been found so far that under the same circumstances could be described by vectors that are neither symmetrical nor antisymmetrical. It is important to

note that none of this could have been deduced from the other standard postulates of Quantum Mechanics. Yet, the Symmetrization Postulate is rarely evoked.

### A. The Spin-Statistics Connection

To determine whether a given particle is a fermion or a boson, we need to investigate its statistical behaviour in the presence of (at least one) other identical particles, when they are all indistinguishable, and this behaviour will be very different for the two types of particles. Indirect methods could also help us reach a conclusion, but before any of that a simple and intriguing property can actually come to our rescue: the *spin-statistics connection*.

**Spin-Statistics Theorem** — *Particles with integer spin are bosons. Particles with half-integer spin are fermions.*

This is not only a widely known empirical rule in Physics, but in fact a theorem (originally proved by Pauli [3]), even if its proofs are not all completely clear and free from controversy. Thanks to it, it is very easy to determine whether some particle is either a fermion or a boson. In particular, this criterion works also for composite particles. It is quite surprising to find such a connection between the spin of a particle and its statistical nature, a connection whose origins I believe are still not well understood.

## III. QUANTUM INFORMATION

The use of quantum systems and their unique properties to encode, transmit, process and store information offers a completely new way to deal with information, representing a revolution for Information Sciences, and possibly for our Information Society as well. It is conceivable that one day we will have a more fundamental description of Nature than Quantum Physics and this may well represent yet again a revolution in the way we deal with information. But before trying to reach that far, we should ask ourselves if we have already explored all the properties of the quantum world in terms of their relevance for information processing. I think not. There is still at least one other property, as fundamental as the ones already mentioned, that should be considered: particle statistics [15], or the apparent fact that every particle is either a fermion or a boson and that their collective behaviour obeys precise rules. Now, can the effects of particle statistics play any role in quantum information processing? Can they be used to perform useful quantum information tasks? And in an efficient way?

For the last couple of years we have been exploring the role of indistinguishable particles and quantum statistics in quantum information processing, both for fermions

and bosons [16]. We have proved that, using *only* the effects of particle statistics, it is possible to perform a quantum information task — such as transfer of entanglement [11], to do useful quantum information processing — such as entanglement concentration [12], and do it in a optimal way — in particular, in a state discrimination protocol [13]. All these results make use the antibunching of indistinguishable electrons impinging in a beam splitter, as well as of the bunching of photons in a similar situation, both a clear signature of their statistics [14].

Using two pairs of entangled particles, it was shown for both fermions (electrons) and bosons (photons) that indistinguishability enforces a transfer of entanglement from the internal to the spatial degrees of freedom without any interaction between these degrees of freedom [11]. Furthermore, sub-ensembles selected by local measurements of the path will in general have different amounts of entanglement in the internal degrees of freedom depending on the statistics of the particles involved. Then, an entanglement concentration scheme was proposed which uses only the effects of particle statistics [12]. Although its efficiency is the same for both fermions and bosons, the protocol itself is slightly different depending on the nature of the particles. Moreover, no explicit controlled operation is required at any stage. Finally, particle statistics is applied to the problem of optimal ambiguous discrimination of quantum states [13]. It was shown that the Helstrom optimal single-shot discrimination probability to distinguish non-orthogonal states of

two qubits (encoded in the internal degree of freedom of two electrons or two photons) can be achieved using only the properties of fermions and bosons. Furthermore, this method offers interesting applications to the detection of entanglement and the purification of mixed states.

Two main features emerge from the above results: particle statistics appears as a resource that can replace controlled operations (conditional interactions) in a *natural* way, and information processing using indistinguishable particles is different for fermions and bosons. The obtained results can also be tested with current technology. Moreover, they establish that indistinguishable particles and quantum statistics can play a new and important role in quantum information and that this connection should be further explored.

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  - [15] Also referred to as *quantum statistics*; I shall use both expressions interchangeably.
  - [16] Note also some recent attempts to use the statistical properties of particles in the context of quantum information using electrons [4, 5], photons [6], parahydrogen [7], fermions [8], bosons [9], and anyons [10], but never presenting a systematic comparison between the fermionic and bosonic statistics (This note and the respective references had to be removed from the published version of this article due to length restrictions).